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MDE/M-23 **4080**

COMPLEX ANALYSIS

Paper–IV, MM–404

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Define Complex line integral and evaluate the integral $\int_{-2+i}^{5+3i} z^3 dz$.

   (b) State and prove Cauchy integral formula for higher order derivatives.

2. (a) State and prove Morrera's theorem.

   (b) State and prove Fundamental theorem of Algebra.

3. (a) State Taylor's theorem. For the function $f(z)=\frac{2z^3+1}{z^2+z}$, find a Taylor's series valid in the neighbourhood of the point z = 1.

   (b) State and prove Minimum modulus principle.

UNIT–II

4. (a) Prove that $\cosh\left(z+\frac{1}{z}\right)=a_0+\sum_{n=1}^{\infty}a_n\left(z^n+\frac{1}{z^n}\right)$ where

   $$a_n=\frac{1}{2\pi}\int_0^{2\pi}\cosh(2\cos\theta)\cos n\theta\, d\theta.$$

   (b) State and prove inverse function theorem.

5. (a) Prove that $\int_0^{\infty}\frac{\log x}{(1+x)^3}dx=-\frac{1}{2}$.

   (b) Show that each analytic function with non-vanishing derivative is conformal in each region.

UNIT–III

6. State and prove Riemann mapping theorem.

7. (a) Define the Gamma function $\Gamma(z)$. Prove that $\Gamma(z+1)=z\Gamma(z)$.

   (b) State and prove Mittag-Leffler's theorem.

8. (a) Define natural boundary. Also prove that unit circle $|z|=1$ is a natural boundary of the function

   $$f(z)=\sum_{n=1}^{\infty}z^{n!}.$$

(b) State and prove Monodromy theorem.

UNIT–IV

9. (a) If G is a bounded Dirichlet region, show that for each $a \in G$ there is a Green function on G with singularity at a.

(b) State and prove Hadamard's three circle theorem.

10. (a) Use Hadamard's factorization theorem to show that

$$\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right).$$

(b) State and prove Bloch's theorem.